CHAPTER 2: FUNCTIONS, EQUATIONS, AND INEQUALITIES

Section 2.1. Linear Equations, Functions, and Models

Solving **linear equations**
Solving word problems: setting up and solving linear models
Ex.: motion problems, simple interest, formulas

**Steps for Problem Solving**
Familiarize, Translate, Carry out, Check, State.

**Concepts**
- **Zero** for a function \( f \): a value \( c \) for which \( f(c) = 0 \)
- **x-intercept**: Point of the form \((c, 0)\) for which \( f(c) = 0 \), point on the \( x \)-axis where the graph crosses or touches the \( x \)-axis
  - \( c \) is a zero for \( f \) \( \iff \) \((c, 0)\) is an \( x \)-intercept

Section 2.2. The Complex Numbers

\[ i = \sqrt{-1} \quad i^2 = -1 \]
Complex number: \( a + bi \) where \( a \) and \( b \) are real numbers
Conjugate of \( a + bi \) is \( a - bi \)
Addition, subtraction, multiplication, and division of complex numbers
  - When dividing by a complex number, to simplify, multiply numerator and denominator by the conjugate of the denominator

Section 2.3. Quadratic Equations and Functions

Quadratic equation: \( ax^2 + bx + c = 0 \) where \( a \), \( b \), and \( c \) are real numbers and \( a \neq 0 \)
Quadratic function: \( f(x) = ax^2 + bx + c \) where \( a \), \( b \), and \( c \) are real numbers, \( a \neq 0 \)
Equation Solving Principles
  - **Principle of Zero Products**: \( ab = 0 \) \( \iff \) \( a = 0 \) or \( b = 0 \)
  - **Principle of Square Roots**: If \( x^2 = k \), then \( x = \sqrt{k} \) or \( x = -\sqrt{k} \)

**Completing the Square**
  - (For \( ax^2 + bx + c = 0 \), start by dividing through by \( a \), take \( b/2 \), square to get \((b^2/4)\) and add to both sides.)

**Quadratic Formula**

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**Discriminant**: \( D = b^2 - 4ac \)
  - \( D = 0 \): one real-number solution;
  - \( D > 0 \): two different real-number solutions;
  - \( D < 0 \): two different complex solutions which are complex conjugates

Equations reducible to quadratic: sometimes possible to do a substitution such as \( u = x^2 \) and reduce an equation to a quadratic equation
Solving word problems involving quadratic models
Section 2.4 Analyzing Graphs of Quadratic Functions

Parabola: graph of a quadratic function
\[ f(x) = a(x - h)^2 + k \]
Graph is the graph of \( y = x^2 \) shifted right by \( h \) units, stretched vertically by a factor of \( a \), and shifted vertically by \( k \) units.
Vertex: \((h, k)\)
Axis of symmetry: Vertical line \( x = h \)
If \( a > 0 \), parabola opens upward, minimum value is \( k \), and range is \([k, \infty)\).
If \( a < 0 \), parabola opens downward, maximum value is \( k \), and range is \((-\infty, k]\).

Graphing quadratic function given in form \( f(x) = ax^2 + bx + c \)

**Approach 1:** Complete the square to put the function in form \( f(x) = a(x - h)^2 + k \), and then read off the vertex, axis of symmetry, and minimum/maximum (depending on sign of \( a \)).

**Approach 2:** Find \( h \) and \( k \) using the formulas: \( h = -\frac{b}{2a} \) and \( k = f(h) \); then write down the vertex, axis of symmetry, minimum/maximum (depending on sign of \( a \)), and form \( f(x) = a(x - h)^2 + k \).

Solving Word Problems: Ex.: Maximizing area, flight of projectile

Section 2.5. More Equation Solving

Solving **Rational Equations:** Be sure to check proposed solutions in the original equation and discard any proposed solution if it results in a zero denominator.

Solving **Radical Equations:**
General strategy: Isolate radical on one side of equation, raise both sides to appropriate power, and simplify. Be sure to check proposed solutions and discard any which do not satisfy the original equation.

**Principle of Powers:** For any positive integer \( n \), if \( a = b \) then \( a^n = b^n \).

Solving **Absolute Value Equations:**
- If \( a > 0 \), \(|x| = a \leftrightarrow x = -a \) or \( x = a \)

Section 2.6. Solving Linear Inequalities

**Principles for Solving Inequalities**
- If \( a < b \), then \( a + b < a + c \)
- If \( a < b \) and \( c > 0 \), then \( ab < ac \)
- If \( a < b \) and \( c < 0 \), then \( ab > ac \)
When both sides of an inequality are multiplied or divided by a negative number, then the direction of inequality reverses.

**Compound inequality:** inequalities joined by “and” or “or.”
- \( a < b < c \) means \( a < b \) AND \( b < c \)

**Absolute Value Inequalities**
- For \( a > 0 \), \(|x| < a \leftrightarrow -a < x < a \)
- For \( a > 0 \), \(|x| > a \leftrightarrow x < -a \) OR \( x > a \)

Solving Word Problems