MATH 107 FORMULAS

Properties of exponents
\[ a^p a^q = a^{p+q} \quad a^p / a^q = a^{p-q} \quad (a^p)^q = a^{pq} \quad (ab)^p = a^p b^p \]

Factoring
\[ (a + b)^2 = a^2 + 2ab + b^2 \quad (a - b)^2 = a^2 - 2ab + b^2 \]
\[ a^2 - b^2 = (a + b)(a - b) \]
\[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \quad a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

Absolute value: \[ |x| = \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases} \]

Distance
The distance between \( a \) and \( b \) on the number line is \(|a - b|\).

The distance between points \((x_1, y_1)\) and \((x_2, y_2)\) is given by \( \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \).

Midpoint Formula: If the endpoints of a line segment are \((x_1, y_1)\) and \((x_2, y_2)\) then the coordinates of the midpoint are \( \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \).

Equation of a Circle:
The equation of a circle with center \((h, k)\) and radius \(r\), in standard form, is \( (x-h)^2 + (y-k)^2 = r^2 \).

Slope
The slope of a line containing points \((x_1, y_1)\) and \((x_2, y_2)\) where \( x_1 \neq x_2 \), is given by \( m = \frac{y_2 - y_1}{x_2 - x_1} \).

Slope-Intercept Equation: \( y = mx + b \), where \( m \) is the slope and \((0, b)\) is the \(y\)-intercept.

Point-Slope Equation: \( y - y_1 = m(x - x_1) \), where \( m \) is the slope and \((x_1, y_1)\) is a point on the line.

Composition of functions: \( (f \circ g)(x) = f(g(x)) \) where \( x \) is in the domain of \( g \) and \( g(x) \) is in the domain of \( f \).

Symmetry of functions
Even function \( \Leftrightarrow \) Graph symmetric with respect to \(y\)-axis \( \Leftrightarrow f(x) = f(-x) \) for all \( x \) in the domain
Odd function \( \Leftrightarrow \) Graph symmetric with respect to origin \( \Leftrightarrow f(x) = -f(-x) \) for all \( x \) in the domain
Transformation of function \( y = f(x) \)

**Vertical translation:**
Graph of \( y = f(x) + b \) is the graph of \( y = f(x) \) shifted **upward** \( b \) units
Graph of \( y = f(x) - b \) is the graph of \( y = f(x) \) shifted **downward** \( b \) units

**Horizontal translation**
Graph of \( y = f(x - d) \) is the graph of \( y = f(x) \) shifted **rightward** \( d \) units
Graph of \( y = f(x + d) \) is the graph of \( y = f(x) \) shifted **leftward** \( d \) units

**Reflection**
Graph of \( y = -f(x) \) is the reflection of the graph of \( y = f(x) \) across the **x-axis**
Graph of \( y = f(-x) \) is the reflection of the graph of \( y = f(x) \) across the **y-axis**

**Vertical Stretching or Shrinking**:
Graph of \( y = af(x) \) is a vertical stretching/shrinking of the graph of \( y = f(x) \)

**Horizontal Stretching or Shrinking**
Graph of \( y = f(cx) \) is a horizontal stretching/shrinking of the graph of \( y = f(x) \)

**Complex number**: \( a + bi \), where \( a \) and \( b \) are real numbers and \( i = \sqrt{-1} \)

**Quadratic Formula**
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

**Quadratic Functions** \( f(x) = ax^2 + bx + c \)
\[
f(x) = a(x - h)^2 + k \quad h = -\frac{b}{2a}, \quad k = f\left(-\frac{b}{2a}\right) = f(h)
\]

**Absolute Value Inequalities**
For \( a > 0 \), \( |x| < a \iff -a < x < a \)  
For \( a > 0 \), \( |x| > a \iff x < -a \) or \( x > a \)

**Polynomials**

**Polynomial Function**: \( P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0 \)

**Intermediate Value Theorem**: For any polynomial \( P(x) \) with real coefficients, if \( P(a) \) and \( P(b) \) have opposite signs, then the function \( P \) must have a real zero between \( a \) and \( b \).

**Remainder Theorem**: Given polynomial \( f(x) \) and value \( c \), when dividing \( f(x) \) by \( x - c \), the remainder is \( f(c) \). That is, \( f(x) = (x - c)Q(x) + R \), where \( R = f(c) \).

**Factor Theorem**: For a polynomial \( f(x) \), if \( f(c) = 0 \), then \( x - c \) is a factor of \( f(x) \).

**Fundamental Theorem of Algebra**: Every polynomial function of degree \( n \), with \( n \geq 1 \), has at least one zero in the system of complex numbers.

**Polynomial with real coefficients** For a nonzero, \( a + bi \) is a zero \( \iff a - bi \) is a zero.

**Polynomial with rational coefficients** For \( a \) and \( c \) rational and \( b \) not a perfect square, \( a + c\sqrt{b} \) is a zero \( \iff a - c\sqrt{b} \) is a zero.

**Polynomial with integer coefficients**

**Rational Zeros Theorem**: If the coefficients are all integers, then for any rational zero (which is a zero of the form \( p/q \)) the numerator \( p \) is a factor of the constant term \( a_0 \) and \( q \) is a factor of the leading coefficient \( a_n \).

**Descartes’ Rule of Signs** The number of positive real zeros of \( P(x) \) is either the same as the number of variations of sign in \( P(x) \) or less than the number of variations of sign in \( P(x) \) by a positive even integer. The number of negative real zeros of \( P(x) \) is either the same as the number of variations of sign in \( P(-x) \) or less than the number of variations of sign in \( P(-x) \) by a positive even integer. A zero of multiplicity \( m \) must be counted \( m \) times.
Rational Function: A quotient of two polynomials, \( f(x) = \frac{p(x)}{q(x)} \), where \( p(x) \) and \( q(x) \) are polynomials and \( q(x) \) is not the zero polynomial.

Direct Variation \( y = kx \), where \( k \) is a positive constant.

Inverse Variation \( y = \frac{k}{x} \), where \( k \) is a positive constant.

Exponential Function with base \( a \): \( f(x) = a^x \), where \( x \) is a real number, \( a > 0 \) and \( a \neq 1 \).

Logarithms
\[ y = \log_a x \quad \iff \quad x = a^y \]

\[ \log_a (MN) = \log_a M + \log_a N \]
\[ \log_a \left( \frac{M}{N} \right) = \log_a M - \log_a N \]
\[ \log_a (M^p) = p \log_a M \]
\[ \log_a 1 = 0 \]
\[ \log_a a = 1 \]
\[ \log_a a^x = x \]