Instructions: You must initialize the Quiz by clicking on the Start button. You may change your answers at any time until you click on End. This will give you your score. The Quiz will not allow to look at answers until you are done with the Quiz. When you are done, press the End Quiz button. Your score will be shown. Then press the correct button. Correct solutions will be checked. Most of the problems have explanations for the solutions, and you can click on the box with the correct answer to see a solution. To return to the Quiz to look at the next problem in the Quiz itself, click on End Quiz at the bottom of the solution. This will return you to where you were.

Start Answer each of the following. Passing is 60%.

1. (2[pts]) Given the set $\{y, \{xy\}, x, \{y\}\}$ we say that y is, in this set, a (or an) :

2. (3[pts]) Given two sets $D = \{xR, s\}$ and $R = \{Y1, x22, pen\}$ of short words (sometimes called strings) how many functions can be defined with D as domain and R as codomain? (Hint: You could just make a list of all of them.)

- 9
- 6
- 10
- 4

3. (3^{pts}) Given the same sets D and R in the previous problem, how many **one-to-one** functions are there with D as domain and R as range? (In fancier language these kinds of functions are called *injections*.

☐ 4 ☐ 2 ☐ 6 ☐ 0

4. (3^{pts}) Given the same sets D and R in the previous problem, how many **onto** functions are there with D as domain and R as range? (In fancier language these kinds of functions are called *surjections.*

☐ 0 ☐ 2 ☐ 4 ☐ 1

5. (2^{pts}) Every function is a relation.

☐ False ☐ True

6. (2^{pts}) Every relation defined as a subset of DXR is a function with domain D and range R.

☐ False ☐ True

7. (5^{pts}) It can happen that for some $x, x = \{x\}$ (Here equals means "same as").

☐ No ☐ Yes

8. (3^{pts}) (Not easy, so be careful) My client has in mind some crazy set. He says the set he has in mind is one which consists of the element 5 and another element which is a set that consists of the element 5 and the set that consists solely of the element 5. Can I write this in my fancy set-theory language I learned in CMSC 150? Which answer below is correct? Recall that for sets, order does not matter.

- ☐ $\{\{5, \{5\}\}, 5\}$
- ☐ $\{\{5\}, \{5, \{5\}\}\}$
- ☐ $\{\{\{5, \{5, \{5, \{5\}\}\}\}\}$
- ☐ None of the above.

9. (2^{pts}) Let $A = \{xy, xx\}$, a set with two words. Is $x \in A$?

☐ No ☐ Yes

10. (3$^{\text{pts}}$) The ordered pair $(3, 2)$ is the same as the set $\{3, 2\}$.

☐ True ☐ False

End Score: Correct

Answers:

Points: Percent:

Letter Grade:

Solutions to Quizzes

Solution to Quiz: Being an **element** or a **member** and being a **subset** are very different ideas. This is one of the the major themes of his little quiz. ■

Solution to Quiz: We can just count them after having made a list. Or, with a little more thought, you can think about it this way: Now, there are two elements in the domain D and three elements in the range R. You can make xR correspond to any of the three elements in R, and for each one of these three choices, you can make the remaining element s correspond also to any of the three elements in R. This gives a total of $3 * 3 = 9$ (3 for the first choice, and for each of these choices, also 3 choices). One such function is: $xR \to pen, s \to pen$. Of course, the number of such functions with any domain of just two elements and range with three elements is **always** 9. ■

Solution to Quiz: Look over your list you made in the previous problem and count them. But a little thought and clever accounting, choose an element in the domain—now you have three possible choices to which it can correspond. But having chosen that element, for the correspondence between the remaining element, you have only two choices left. Hence the answer is $3 * 2 = 6$. The example of a function in the previous problem is NOT one to one. If you like the machine description of functions, for distinct things the machine produces distinct results.

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Solution to Quiz: Look over the solution to the first problem. After you have chosen values in the range for the two elements in the domain you always have an element in the range that does not correspond to anything in the domain. So there are no onto functions. If still in doubt, try to find one. Put two things on a table, and three things on a table. Try to draw arrows from the two-thing set to the three-thing set. If you succeed, you will notice you drew two arrows from an element in the two-set. This is NOT a function since it requires that for each element in the domain there be ONE AND ONLY ONE element in the range to which it corresponds. But the correspondence you drew is a more general structure called a RELATION.

■

Solution to Quiz: In order for something to be a relation here, it must be a subset of DXR the Cartesian product of some two sets. Given any function f with domain D and range R. The set of ordered pairs for all $x \in D$ of the form $(x, f(x))$ (sometimes this is called the GRAPH of the function and it completely determines what the function does, that is $x \rightarrow f(x))$ is a subset of DXR. Therefore, any function can be considered a relation. Just imagine it described completely by its graph.

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Solution to Quiz: Intuitively it might be clear that this is not possible since you can have multiple things in R corresponding in a single element in D. Think of two arrows going from one element of the domain to more than one in the range. This is a no-no. It is like a machine given the same materials that sometimes produces a pot and at other times produces a shoe. But a counter example will show this perhaps more decisively. Let the relation be "nephew" between D and R both being the set of all persons, so for a person $x\ inD$ and $y \in R$ the relation will be spoken as: "x is a nephew of y". This is not a function since x could have many uncles, that is there is no necessarily unique correspondence between a given x and a single uncle y. If you are fond of more formal counterexamples in the style that mathematicians like, let $D = /a/R = /1, 2/$ and let the relation be defined by ALL of DXR. This means the ordered pair $(a, 1)$ and $(a, 2)$ are both in the relation, so if it were a function say f, then $f(a) = 1$ but also $f(a) = 2$. Recall what you did when graphing functions in your algebra classes. The graph of a function means any vertical line from the x-axis in the domain of the function intersects the graph in only one point. Notice that a circle on plane with a coordinate system does NOT define a function, but it is a relation: xRy means $x^2 + y^2 = 1$.

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Solution to Quiz: A very important distinction is being made here, and it requires some careful reading. Ask yourself what do you do when you take some objects, say Jim, x, and Charlie, and form the set of these objects? The SET of these objects is VERY VERY DIFFERENT from Jim, x, and Charlie. When we form the set of them, you are GATHERING them together as if they BELONGED together. Perhaps they are all simply things, or maybe just persons, somebody called Jim, somebody unknown we call x, and somebody else we call Charlie. Whatever (as we say these days). Consider the number 1. Now consider the set consisting of the number one $\{1\}$. Ask yourself this: **What are you doing when you place those curly brackets around some ”list” of objects?** Jim, x, and Charlie has nothing special about it in ordinary language. But when I say I want you to think about Jim, x, and Charlie TOGETHER, what am I doing? My suggestion is that you are forming a purely MENTAL CONSTRUCT, belonging entirely to the WORLD OF ABSTRACTION, when you form the set consisting of the elements Jim, x, and Charlie. In this act, you will have declared you are a separate being from the beasts!

Speaking “computerease” think of Jim, x, and Charlie just on a computer stack. They are not together in any way until I GATHER them into some array or process them in some way.

Conclusion: The **concept of the object** is NEVER the same as the object. So, $\{Ann\} \neq Ann$, because on the left is an object, but on the right is a MENTAL CONSTRUCT related to but not the SAME AS the object; this construct is called the SET consisting of the element which element is the object Ann. Please try to be clear about this distinction. We must try to be clear why the answer is 'No'. When you put brackets around something, it is an act PURELY IN YOUR HEAD! Take a piece of paper. Write something on it. Now put brackets around it. What have you done? This experience, however, is only for you. When the computer reads the brackets, it does not care what you think, but it knows what to do. You too should know what it does and that the computer's work is different from our own. ■

Solution to Quiz: In a paper I posted Ms. Epp, the author of the CMIS160 text, wrote a paper about how hard this course was to teach. (See the Webliography.) She wrote that students who do poorly have a 'linguistic problem'; surely we can read this problem and translate it, no? No linguistic problem here! Start with the innermost parentheses, the set that consists of the element 5, next the set that consists of that set and the element 5, and next, the last set, is the set which consists of that set and the element 5. Each time, you will add a bracket. Is it just a matter of counting? I often miss a parenthesis myself. How do you do it? The computer has no problems. ■

Solution to Quiz: I hope no one is confused with this. The set consists of words, and has only two elements. x is part of a word which word is an element of A, but not itself a member of A. A word is a list (because order is important) and we can form a set of lists, or s lists of sets.

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Solution to Quiz: An ordered pair or an n-tuple is not a set since it has a lot more structure, in particular, order. Strangely, the project of defining an ordered pair from set theory gave mathematicians a lot of pain. This is not our project. Somehow lists seem much more intuitive for us and easier to implement than sets, so we could define a set as a list where the order is not important. If you get the even intuitive sense that a set of two elements, say $\{3, 2\}$ is very different from the coordinates of a point $(3, 2)$ you have all you need to know at this point (excuse pun!). The problem, I think, is when we write out a set, we impose a kind of order on it which is not there, as when I write x, 56, 9. If I mean this as a set, then the order is not part of the structure. ∎