UMUC

CMSC150 Self-Grading Quiz: Review

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Instructions: You must initialize the Quiz by clicking on the Start button. You may change your answers at any time until you click on End. This will give you your score. The Quiz will not allow to look at answers until you are done with the Quiz. When you are done, press the End Quiz button. Your score will be shown. Then press the correct button. Correct solutions will be checked. Most of the problems have explanations for the solutions, and you can click on the box with the correct answer to see a solution. To return to the Quiz to look at the next problem in the Quiz itself, click on End Quiz at the bottom of the solution. This will return you to where you were.

Start Answer each of the following. Passing is 60%.

1. The negation of the statement $(\forall x \in R,\ x^2 = 2)$ is

- ☐ The square of all real numbers is 2.
- ☐ Some real numbers have square 2
- ☐ No real number has square 2
- ☐ There exists a real number x such that $x^2 = 2$
- ☐ There exists a real number such that $x^2 \neq 2$

2. How many elements are in the union of four sets if each of the sets has $a = 96$ elements, each pair of sets share $b = 47$ elements, each triple of sets shares $c = 25$ elements and there are $d = 4$ elements in all four sets?

3. The following sequence $s_n = (-1)^n \;\; n \geq 1$ satisfies the recurrence relation for all integers $k \geq 1$:

- $s_k = (-1)^k s_{k-1}/k$
- $s_k = (-k)s_{k-1}$
- $s_k = ks_{k-1}$
- $s_k = (-1)^k k s_{k-1}$

4. Consider the statements:
Everyone good at mathematics is happy.
Charles is not happy.
Therefore, Charles is not good at mathematics.
Is this inference valid?

☐ Yes ☐ No

5. Consider once more the statements:
Everyone good at mathematics is happy.
Charles is not happy.
Therefore, Charles is not good at mathematics.
What are all the rules you must use, other than the hypothesis, to make this inference valid?

- ☐ mt,ei, ui
- ☐ mt
- ☐ mt, ei
- ☐ ei, ui

6. To prove the conjecture "If Laramie is the capital, then Wyoming is the state," it is sufficient to prove "If Wyoming is not the state, then Laramie is not the capital."

☐ True ☐ False

7. A collection V of numbers is defined recursively by:

1. 6 and 8 belong to V
2. If X and Y belong to V, then so does $X + 2Y$

Does every even number ≥ 18 belong to V?

☐ Yes ☐ No

8. Every infinite set is denumerable.

☐ True ☐ False

9. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 995 elements in A_2 and 10010 elements in A_3 in following situation: **The sets are pairwise disjoint.** Do not use commas in writing your answer.

10. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 995 elements in A_2 and 10010 elements in A_3 in following situation: $A_1 \subseteq A_2 \subseteq A_3$ Do not use commas in writing your answer.

11. Find the number of elements in $A_1 \cup A_2 \cup A_3$ if there are 100 elements in A_1, 995 elements in A_2 and 10010 elements in A_3 in following situation: **There are** 15 **elements common to each pair of sets and** 4 **elements in all three sets.** Do not use commas in writing your answer.

12. There are 5 different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

13. Is the empty set a proper subset of every set?

☐ Yes ☐ No

14. True or False: $\emptyset \in \{\emptyset, \{\emptyset\}\}$

☐ True ☐ False

15. True or False: If A and B are disjoint sets, then $(A-B)\cup(B-A) = A \cup B$

☐ True ☐ False

16. Given $|A| = 3, |B| = 6$, then $|AXB|$ has

[ ] elements.

17. If P is a sufficient condition for Q, then Q is a necessary condition for P.

☐ False ☐ True

End Correct

Answers:

Points: Percent:

Answers:

Solutions to Quizzes

Solution to Quiz: 1.

This is an application of the wff at the top of page 57.

Solution to Quiz: 2.

Inclusion-Exclusion Principle which gives: $4a - 6b + 4c - d = 198$.

Solution to Quiz: 3.

A little algebra here:

$$s_k = (-1)^k k! = (-1)(-1)^{k-1} k(k-1)! = (-1)s_{k-1}$$

Solution to Quiz: 4.

What kind of inference rule will you use, *modus tollens* or *modus ponens*? What other rules must be used here? See the next problem.

Solution to Quiz: 5.

See Example 27. It is a deduction involving quantifiers like this question, except we use mt, *modus tollens*, rather than *modus ponens*.

Solution to Quiz: 6.

The two statements are contrapositives of each other and so are logically equivalent. See also 4, page 181.

Solution to Quiz: 7.

Applying the recursive definition, we quickly see that, 6, 8, 18, 20, 22, 24, are the only numbers in V less than 26. If 26 is in V, it must be written as X + 2Y where X and Y are any two numbers in this list. Since X must be greater than or equal to 6, Y must be either 6 or 8, otherwise X + 2Y will be greater than 26. So we have two cases: X + 12 = 26 or X + 16 = 26, or either X = 14 or X = 10, neither of these numbers is in the list. So, 26 is not in V. This is problem 17, on page 182.

Solution to Quiz: 8.

The Cantor diagonalization shows that the real numbers, an infinite set, is NOT denumerable.

Solution to Quiz: 9.

Being all pairwise disjoint, the total number of elements is simply the sum of the elements in each of the sets. ■

Solution to Quiz: 10.

Since all the sets are contained in A_3, the answer is just $|A_3|$. ■

Solution to Quiz: 11.

$$9969100 + 995 + 10010 - (3 * 15) + 4 = 11064$$

Solution to Quiz: 12.

$5! = 120$.

Solution to Quiz: 13.

The key word here is **proper**. There is one case where it is NOT a proper subset, $\emptyset \subseteq \emptyset$. On the other hand, this statement is false $\emptyset \subset \emptyset$ because $\emptyset = \emptyset$. But for any set A, it is false that $A \subset A$. See Practice 4, page 189.

Solution to Quiz: 14.

Yes, sets can be members of sets. This is an example. ■

Solution to Quiz: 15.

If A and B are disjoint sets, then A-B = A and B-A = B. ■

Solution to Quiz: 16.

This is an application of the multiplication rule. ■

Solution to Quiz: 17.

This is just page 4 again.