THE UNIVERSITY OF MARYLAND
CMSC150

Posets and Graphs Card Drill

R. T. Harris

Instructions: Click on the Begin button to view the first randomly selected card. Click on FS to view the flash cards in full screen mode (works only outside a web browser). The Home button will bring you to my website. The Close button closes the document (use outside a web browser).

Last Revision Date: June 12, 2008

rth0@comcast.net

QUESTION

The set of integers with the relation $<$ is a poset.

(a) True
(b) False

QUESTION

Does there exist a tree with twelve vertices and fifteen edges?

(a) True
(b) False

QUESTION

Because a tree is a graph, a complete tree is also a complete graph.

(a) True
(b) False

QUESTION

The *degree* of a vertex of a graph is the number of [   ] that are incident on it.

QUESTION

There exists graphs of degree 11.

(a) True
(b) False

QUESTION

Let $\Sigma = \{a, b\}$ and let R be the following partial ordering on Σ:

$$R = \{(a,a), (a,b), (b,b)\}$$

And denote the lexicographic ordering (see page 241) for Σ^* corresponding to R by $\succeq$. Is $aba \succeq babaaa$?

(a) True
(b) False

QUESTION

Totally ordered sets are basically just like a single tower where the elements are stacked on top of each other.

(a) False
(b) True

QUESTION

We know that every graph is a mapping from a set of vertices to a set of edges, and this mapping is called the *edge-endpoint function*–see the definition on page 404. The *edge-set* is simply a set of either two or one vertices. Consider the following graph with three edges e_1, e_2, e_3 and five vertices $v_1 \ldots v_5$. Let the edge-endpoint function be: $f(e_1) = \{v_1, v_2\}, f(e_2) = \{v_1, v_3\}, f(e_3) = \{v_4, v_5\}$. How many components does this graph have?

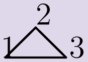
QUESTION

True or false: The power set $\mathcal{P}(\{a, b, c\})$ under the subset order relation is a totally ordered set.

(a) True
(b) False

QUESTION

Write for yourself the adjacency matrix of the following undirected graph:



Is this matrix a symmetric matrix?

(a) True
(b) False

QUESTION

Given any adjacency matrix A. Multiply it with itself. Take an entry from this matrix, say a_{ij}. Then a_{ij} is the number of possible paths from the ith vertex to the jth vertex.

(a) False
(b) True

QUESTION

Draw the graph whose adjacency matrix is:

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

HINT

Are you sure all the conditions for being a poset are met?

ANSWER

The ordering is not reflexive, so it is not a poset. With the relation 'less than or equal' the integers are a poset.

HINT

Is there some theorem about the relation between the number of vertices and the number of edges for trees?

Hint Soln
Next Home

ANSWER

Yes, there is some theorem, and it says there is no such beast.

HINT

Look up the definition.

ANSWER

See the sentence at the end of the text on page 434.

HINT

Just look up the definition

ANSWER

If a vertex that has three loops on it and no other edge connecting it, do you see what it's degree would be?

HINT

Try drawing a graph of degree 3.

ANSWER

It is false. Can you visualize why this is so? Draw a random graph with four edges. Now add up all the degrees of the vertices. Do you see how they relate to the number of edges? How much does each edge contribute to the total degree?

HINT

Recall how you look things up in a dictionary.

ANSWER

I hope this is obvious.
Now consider what happens to the problem if I omitted the ordered pair (b, b) in the relation? What would happen if I omitted (a, b)? In this relation, is $\in \succeq aabb$?

HINT

What I call a tower here is what your text calls a chain.

ANSWER

The numbers either all integers or real numbers or rational numbers are all examples of totally ordered sets. The Hasse diagram of a totally ordered set is a single line with the vertices one upon the other.

HINT

Draw the graph first. Please recall there are many pictorial representations of the same graph.

ANSWER

This graph has only two components. One component has the edges e_1, e_2 and the other component consists of the single edge e_3.

HINT

Is it really true that for any pair of subsets in this power set of three elements, for any pair of subsets, one must be a subset of the other?

ANSWER

False. Consider any two singleton subsets, say $\{a\}$ and $\{b\}$ where it is clear that neither is the subset of the other. Draw the Hasse diagram.

HINT

Recall that any entry in the ith row and the jth column gives the number of arrows *from* the ith vertex to the jth vertex.

ANSWER

If the number in the i,j slot is the same as the number in the j,i slot (that is, the matrix is symmetric along the diagonal, then we can go in either direction from those two vertices, so the graph is sometimes called *bidirectional*. If those were were different, then there would be an edge on which you could go, but could not return along that same edge. The adjancency matrix for this graph is pretty simple: $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

HINT

Look up the reachability matrix.

ANSWER

The i,jth entry of the product of A with itself gives the number of walks OF LENGTH 2 ONLY from the ith vertex to the jth vertex. The proof of this theorem uses a proof by induction for general n which methods you have not yet seen, but the proof for the case n = 2 does not need induction.

HINT

If possible you must go back and see how these adjacency matrices are constructed. You are learning how to translate the matrix into a graph and the graph into a matrix. Such matrices are the usual ways graphs are entered into a computer.

ANSWER